Investigation 2

ACE Assignment Choices

Differentiated Instruction Solutions for All Learners

Problem 2.1

Core 1, 2, 4, 15–16, 21
Other *Applications* 3, *Connections* 17–20, *Extensions* 31–33

Problem 2.2

Core 5, 6, 8
Other *Applications* 7, *Connections* 22, 23; unassigned choices from previous problems

Problem 2.3

Core 9–13, 24–27
Other *Applications* 14; *Connections* 28–30; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 3 and other ACE exercises, see the *CMP Special Needs Handbook.*
Connecting to Prior Units 21: *Prime Time;* 22, 24–27: *Moving Straight Ahead;* 23, 28, 29: *Stretching and Shrinking;* 30: *Bits and Pieces I*

Applications

1. **a.** $b = 4^n$

   b. $4^7 = 16{,}384$ bacteria

   c. 65,536; this can be found by computing $16{,}384 \cdot 4$ because $4^8 = 4^7 \times 4$.

   d. 10 hours. There will be at least 1 million bacteria in the colony after 9 hr and before 10 hr, as shown by $4^9 = 262{,}144$ and $4^{10} = 1{,}048{,}576$. (Note: This is essentially solving the equation $1{,}000{,}000 = 4^n$. Students can solve this problem in a variety of ways. They might guess and check values of n in 4^n. They might make a chart. They might enter the equation into a calculator and look at the table. They might trace a calculator graph, although setting an appropriate graphing window for exponential equations can be challenging.)

   e. $b = 50(4^n)$

   f. There will be 13,107,200 bacteria after 9 hr and 52,428,800 after 10 hr. We can find these by multiplying the number of bacteria at hour 8 by 4, and then multiplying that number by 4.

2. **a.** Growth of Loon Lake Plant

Year	Area Covered (sq. ft)
0	5,000
1	7,500
2	11,250
3	16,875
4	25,312.5
5	37,968.75

   b. 10 yr (more closely, about 9.098 yr)

3. **a.** Leaping Leanora's Salary

Year	Salary
1	$20,000
2	$40,000
3	$80,000
4	$160,000
5	$320,000
6	$640,000
7	1,280,000
8	$2,560,000
9	$5,120,000
10	$10,240,000

   b. $20,460,000. (Note: Students can find this by adding the amounts in the table or by using their calculators to find the sum of the sequence of $S = 10{,}000(2^n)$ from $n = 1$ to 10. See page 16 for information on how to do this.)

c. The growth pattern is doubling from year to year.

d. $s = 20{,}000(2^{n-1})$ or $s = 10{,}000(2^n)$

4. a. 25 beetles; 35 beetles; 45 beetles

b. 45 beetles; 135 beetles; 405 beetles

c. $b = 5 + 10m$, where b is the number of beetles and m is the number of months

d. $b = 5(3^m)$ or $b = 15(3^{m-1})$, where b is the number of beetles and m is the number of months

e. 19.5 months. Solve $200 = 5 + 10m$.

f. Between 3 and 4 mo. There are 135 beetles after 3 mo and 405 beetles after 4 mo. (**Note:** Students won't be able to solve the exponential equation algebraically. They can find an approximate solution by scrolling through a calculator table for the equation, using appropriate increments. Or, students might graph the equation and trace its graph.)

5. a. 60; the number of fruit flies in any generation divided by the number in the previous generation is 60.

b. 1,555,200,000; $432{,}000 \times 60 \times 60 = 1.5552 \times 10^9$

c. $p = 2(60^g)$

d. 4

6. a. 12 mice. There were 36 mice after 1 mo and the growth factor is 3. So, at 0 mo, there were $36 \div 3 = 12$ mice.

b. $p = 12(3^n)$. 12 is the original population, 3 is the growth factor, p is the population, n is the number of months. [Or, $p = 36(3^{n-1})$, where 36 is the population after 1 mo.]

7. a. 8 fleas

b. 3

c. $8(3^{10}) = 472{,}392$ fleas

Note to the Teacher Point out to students that this answer demonstrates that exponential growth equations have limits as models of real-life phenomena. Although something might start out increasing in a nearly exponential way, the predictive validity of the model will eventually break down.

8. a.

x	y
0	150
1	300
2	600
3	1,200
4	2,400
5	4,800

b. The starting population, or initial value, is 150, and the growth factor is 2.

9. Growth factor: 3; y-intercept: 300

10. Growth factor: 3; y-intercept: 300

11. Growth factor: 2; y-intercept: 6,500

12. Growth factor: 7; y-intercept: 2

13. a. After 2 yr, the lizard population was 40.

b. After 1 yr, the lizard population was 20.

c. Between years 3 and 4.

d. Divide the population for one year by the population for the previous year. For example, divide the population for year 3, which is 80, by the population for year 2, which is 40: $80 \div 40 = 2$.

14. a. The growth factor for Species X is 3 because the y-value for each point is 3 times the previous y-value. The growth factor for Species Y is 2 because the y-value for each point is 2 times the previous y-value.

b. The y-intercept is (0, 5), so the starting population for Species X is 5.

c. The y-intercept is (0, 25), so the starting population for Species Y is 25.

d. $y = 5(3^x)$

e. $y = 25(2^x)$

Connections

15. D

16. G

17. 4.88×10^7

18. Less than; 1 million is 10^6 and $3 < 10$. Therefore, $3^6 < 10^6$.

19. Less than; 1 million is 10^6 and $9 < 10$. Therefore, $9^5 < 9^6 < 10^6$.

20. Greater than; 1 million is 10^6 and $10 < 12$. Therefore, $10^6 < 12^6$.

21. **a.** $3^2 \times 5$

b. $2^4 \times 3^2$

c. $2^3 \times 11 \times 23$

22. **a.** The y-intercept is (0, 10) for each equation.

b. If you make a table of (x, y) values for Equation 1 for consecutive x-values, you will see that the y-values decrease by 5, so the rate of change is -5. In the table for Equation 2, the values increase. If you subtract successive y-values, you get differences of 40, 200, 1,000, and so on. So the rate of change is increasing. (Students will learn in Investigation 3 that the growth rate is 400%.) (Note: Students may describe the pattern of change for Equation 2 multiplicatively, saying that each y-value is 5 times the previous y-value. You could ask these students to describe the change additively, which will get at the increasing rate of change described above.)

c. In Equation 1, the rate of change (the slope) is the -5 in front of the x. In the Equation 2, the rate of change is harder to see. It is easier to see in a table. However, the growth factor of 5 can be seen in the equation as the number raised to the exponent. (Note: Students will be introduced to rate of change of exponential equations in Investigation 3, so this problem is just to get them to think about patterns of change for linear and exponential functions.)

d. Look at the vertical distance between points for each horizontal change of 1 unit. In the graph of Equation 1, the vertical distance between any two points is 5. In the graph of Equation 2, the vertical distance increases, indicating that the y-values are increasing at a faster and faster rate. Students may also describe this multiplicatively at this time.

23. **a.** (Figure 1)

b. Exponential; each perimeter is multiplied by 2 to obtain the next perimeter.

c. Exponential; each area is multiplied by 4 to obtain the next area. (Note: Because both width and length increase by a factor of 2, area increases by a factor of 4.)

d. $P = 10(2^n)$

e. $A = 6(4^n)$ or $A = 3(2^n) \times 2(2^n)$

f. Perimeter and area would still increase exponentially, but the related equations would be $P = 10(3^n)$ and $A = 6(9^n)$.

24. $y = \frac{1}{4}x + 4$; slope is $\frac{1}{4}$, y-intercept is (0, 4)

25. $y = 2x - 6$; slope is 2, y-intercept is (0, -6)

26. $y = -3x - 3$; slope is -3, y-intercept is (0, -3)

27. $y = 3$; slope is 0, y-intercept is (0, 3)

28. Ahmad. Expressed as a percent, Kele's scale factor is 200%, which is less than 250%.

29. C

30. Gizmo seller, gadget inspector, widget designer

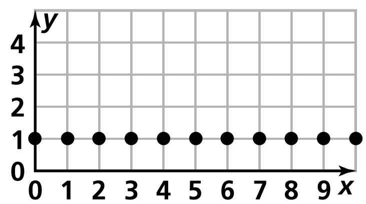
Figure 1

Enlargement	Dimensions (cm)	Perimeter (cm)	Area (cm^2)
0 (original)	2 by 3	10	6
1	4 by 6	20	24
2	8 by 12	40	96
3	16 by 24	80	384
4	32 by 48	160	1,536
5	64 by 96	320	6,144

Extensions

31. a.

x	y
0	1
1	1
2	1
3	1
4	1



b. The equation $y = 1^x$ looks like other exponential equations, but the pattern in the table—in which every value of 1^x is 1—and in the straight-line graph looks like a linear relationship.

32. a. $y = 3(2)^x$; the growth factor can be found by dividing the y-values: $12 \div 6 = 2$. The y-intercept can be found by dividing the y-value for $x = 1$, which is 6, by the growth factor of 2. So the y-intercept is (0, 3).

b. $y = 10(3)^x$; the growth factor can be found by dividing the y-values: $270 \div 90 = 3$. The y-intercept can be found by dividing the y-value for $x = 1$, which is 30, by the growth factor of 3. So the y-intercept is (0, 10).

33. a. Dawn. At the end of 20 years, Leonora would have \$1,000,000(20) = \$20,000,000, and Dawn would have $2^{20} - 1$ = \$1,048,575. Students will probably sum up the values for each year to find Dawn's total: \$1 + \$2 + … + $\$2^{19}$ = \$1,048,575. (Note: Dawn receives 2^{n-1} dollars in salary, where n is the year number, and her total for the n years is $2^n - 1$.)

b. Leonora will continue to have a greater salary through year 25, when Dawn will overtake her with \$33,554,431 to Leonora's \$25,000,000.

Note to the Teacher: You may want to discuss with students a realistic time span for players in professional basketball. It is unusual for players to remain in high demand for 20 years or more. Salaries may even decrease after time.

Possible Answers to Mathematical Reflections

1. a. Using the table, the y-intercept is the point where $x = 0$. If the y-intercept is not given in the table, you can use the growth factor to find it. To find the growth factor, divide a y-value by the previous y-value. Then, start with a y-value and divide by this growth factor, moving backward in the table until you find the y-value for the point with x-coordinate 0. Using the equation $y = a(b^x)$, the y-intercept is a and the growth factor is b. Using the graph, the y-intercept is the point where the graph crosses the y-axis. The growth factor can be found by taking two points on the graph with x-values 1 unit apart, such as (2, 4) and (3, 8) and then dividing the second y-value by the first y-value.

b. If a is the y-intercept and b is the growth factor, then write the equation $y = a(b^x)$.

2. a. a is the y-intercept (the value when $x = 0$), and b is the growth factor.

b. a is the y-intercept.

c. In the graph of $y = a(b^x)$, the b is how much one y-value is multiplied by to get the next y-value. A greater value of b will increase the rate of growth, resulting in a steeper graph at each x-value.